

# The vertical equilibrium of molecular gas in the galactic disk

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## ABSTRACT

We examine the vertical structure and equilibrium of the molecular gas layer in the galactic disk, measuring its scale height and velocity dispersion as a function of Galactic radius by modeling the CO emission at the tangent points.

The model takes into account emission from a large path length along the line of sight, corresponding to an interval ( $\Delta R$ ) of typically a few hundred parsecs in galactic radius; and is parametrized by the scale height of the gas, the centroid in  $z$ , the rotation velocity and the velocity dispersion; these parameters are assumed to be constant over the interval  $\Delta R$ . This model is then fit to the  $^{12}\text{CO}$  survey of Knapp et al.(1985) to determine the best fit parameters.

The terminal velocity values are found to be in good agreement with those obtained from HI data. The  $\chi^2$  per degree of freedom from fitting the models (considering only the photon statistics) range from 2 to 19, with a median value of 6. The main source of error is found to be the ‘shot noise’ due to the small number of clouds. Simulations of the tangent point emission using discrete molecular clouds are carried out to estimate the errors. We use the observed radial distribution of molecular gas and the ‘standard’ size-linewidth relation for molecular clouds in the simulations. Modeling the simulations gives values of  $\chi^2$  per degree of freedom similar to those obtained from modeling the observations.

The scale height of the distribution is found to increase fairly monotonically with radius. The value of the velocity dispersion varies from  $\sim 2 \text{ km s}^{-1}$  to  $11 \text{ km s}^{-1}$  with a typical uncertainty of  $\sim 3 \text{ km s}^{-1}$ . The variation in velocity dispersion is consistent with a monotonic increase with galactic radius.

The midplane mass density of the disk  $\rho_0(R)$  is calculated from the scale-height and velocity dispersion (under the assumption that the velocity dispersion is isotropic) and is consistent with the local value  $\rho_0(R_0)$  of  $0.2 M_\odot \text{ pc}^{-3}$  determined from stellar kinematics.

*Subject headings:* interstellar gas, Galaxy, internal motions, mass models

## 1. INTRODUCTION

Because of its expected simplicity, the distribution and kinematics of gas in the direction perpendicular to the plane of the disk of the galaxy should have much to tell us about the forces acting on it and its state of equilibrium. Considering the most obvious force, gravity, and recalling that the scale height of molecular gas is much smaller than that of stars, we expect a Gaussian distribution in  $z$   $\rho(z) = \rho_0 \exp(-z^2/2\sigma_z^2)$  (Spitzer 1942), if the velocity dispersion  $\sigma_v$  of the gas is constant. The Gaussian scale height is then  $\sigma_z^2 = \sigma_v^2/4\pi G\rho_0$ , where  $\rho_0$  is the mid-plane mass density of the disk.

Knowing the scale height and the velocity dispersion in the vertical direction one can estimate  $\rho_0$ , the mid-plane mass density of the disk. Studies using stellar populations (F dwarfs and K giants) as test particles to probe the potential give estimates of both the local disk mass density  $\rho_0$  and the integral surface mass density  $\Sigma_0$  (Bahcall et al. 1992, Bahcall 1984a, Bahcall 1984b, Bahcall 1984c, Bienaymé et al. 1987, Kuijken & Gilmore 1989a, Kuijken & Gilmore 1989b, Kuijken & Gilmore 1989c, Kuijken & Gilmore 1991a, Kuijken & Gilmore 1991b, Kuijken 1991). In a complimentary approach we can hope to probe the potential at different galactocentric distances with gas (clouds) since gas emission from the whole disk can be observed. Since the gas disk is much thinner than the stellar disk, its vertical distribution probes only the midplane mass density. A similar study has been carried out for HI by Celnik et. al (1979) and Merrifield (1992).

The kinematic information about molecular clouds, especially their velocity dispersion is important for determining their dynamics, with each other as in collisions (influencing cloud formation/destruction, star formation); and with the stars, increasing the velocity dispersions of stars (Spitzer & Schwarzschild 1951). The variation of velocity dispersion with Galactic radius may give clues to the sources of random kinetic energy to the clouds (Gammie et al. 1991), star/cloud formation as a function of galactic radius etc.

Since the molecular gas forms a cold, thin layer in the disk, it may be possible to look for signatures of disequilibrium in departures from Gaussianity of the distribution. These departures may be due to violent events stirring up the gas (for example supernovae), short lifetimes of molecular clouds, infalling gas etc. or because of more than one isothermal population with different values of velocity dispersion  $\sigma_v$ .

The large scale distribution of molecular gas has been mapped using the Massachusetts-Stony Brook galactic plane survey of  $^{12}\text{CO}$  in the plane by Sanders et al. 1986, Clemens et al. 1986; and the Columbia survey by Cohen et al. 1986, Bronfman et al. 1989, Dame et al. 1987. We use data from the Bell Labs  $^{12}\text{CO}$  survey (Knapp et al. 1985), consisting of strip maps in latitude because we need good latitude coverage and sacrifice close sampling

in longitude. The details of the survey and the data as they relate to the present analysis are presented in section 2.1.

The scale height of the molecular gas has been previously estimated by Clemens et al. (1988) to be about 65 pc at  $R = 0.95R_0$  and by Sanders et al. (1984), Bronfman (1988), to be about 60 pc. Different surveys thus give very consistent values of the scale height.

The same cannot be said for the velocity dispersion  $\sigma_v$  of the clouds. The values of  $\sigma_v$  found by different groups are inconsistent by a factor of two, if not more. The velocity dispersion for local (distance from the sun  $< 3$  kpc) clouds has been estimated with the additional input of their distances and the Galactic rotation curve by Stark (1984) and Stark and Brand (1989), to be  $7.8^{+0.6}_{-0.5}$  km s $^{-1}$ . Blitz et al. (1984) find  $\sigma_v = 5.7 \pm 1.2$  km s $^{-1}$  for high latitude (hence local) clouds. Liszt and Burton (1983) estimate a single value of  $\sigma_v = 4.2$  km s $^{-1}$  for the inner galaxy by measuring the dispersion of what they call ‘jitter’, which is the difference between the terminal velocity of HI and that of  $^{13}\text{CO}$ . Clemens (1985) estimates a similar value by fitting a Gaussian to the tangent point emission. To get rid of emission coming from galactic radii smaller than the tangent point radius (sub-tangent point emission) the fit is restricted to  $\pm\sigma_z$  since the emission from nearby gas extends to higher latitudes. The present paper takes a new approach to modeling the tangent point emission by fitting simultaneously the terminal velocity, velocity dispersion and the scale height to the observations. We do not try to isolate the tangent point emission but take into account emission from the sub-tangent point gas. The modeling procedure is described in sections 2.2 and 2.3.

We use this model to get the best fit values of the aforementioned parameters for each longitude observed. As expected the main source of uncertainty in the parameters is due to the clumpiness of the molecular gas distribution. To estimate these uncertainties in the parameters measured, we carry out Monte Carlo simulations of the cloud distribution in the galaxy, using the known radial distribution of gas and standard cloud properties. The details of the simulations are discussed in section 2.5.

In section 3 we present the best fit parameters, and try to interpret their variation across the inner galaxy. The errors and systematic shifts in these parameters are estimated from the Monte Carlo simulations. We also estimate the midplane mass density of the disk.

Section 4 contains a discussion of different values, methods and the definitions of the parameters characterizing the galactic distribution as estimated by previous studies; and how consistent various determinations are, particularly with respect to velocity dispersion estimates. The assumptions behind this study and various caveats are also touched upon. The conclusions of this paper are summarized in section 5.

Appendix A contains contour maps of CO emission as a function of latitude and velocity, for several different longitudes. Contours of emission according to the best fitting models are superposed.

The value of the circular velocity at the Sun  $\Theta_0$  is taken to be  $220 \text{ km s}^{-1}$ , and the assumed distance to the galactic center is 8.5 kpc.

## 2. METHOD

### 2.1. Data

The observations analyzed here were made using the 7 meter antenna at Bell Laboratories. The data and details of the survey and the instruments are presented by Knapp et al. (1985). Here we summarize the information necessary for the present analysis.

The survey consists of 38 strip maps (in latitude) of the  $^{12}\text{CO}(1 \rightarrow 0)$  line emission. The maps are sampled at intervals of  $\Delta b = 2'$  with a half-power beamwidth of  $100''$ . The latitude coverage is  $\simeq \pm 2^\circ$ , varying slightly from one line of sight to another. The latitude extent is more than adequate for studying the tangent point emission (except possibly for  $l \geq 60^\circ$ ), extending more than three scale heights both above and below the centers of distribution in more than half the lines of sight.

The strip maps were taken at longitudes between  $4^\circ$  and  $90^\circ$ , spaced at equal intervals of  $\Delta \sin l = 0.025$ . The velocity resolution is  $0.65 \text{ km s}^{-1}$ . The rms noise is typically 0.3 K but varies slightly in each latitude strip. Linear baselines were removed from individual spectra. Exceptions to this treatment are few and are listed by Knapp et al. (1985).

We are able to analyze 22 of the 38 observed lines of sight. The low longitude ( $4^\circ < l < 17^\circ$ ) maps were unusable due to the lack of emission at (or reasonably near) the expected tangent point velocities. The high longitude ( $l > 61^\circ$ ) maps were unusable because their tangent point emission is at low velocities and is contaminated by emission in reference positions; and because the latitude extent of the maps is not adequate for this (almost) local emission.

### 2.2. Tangent points

Assuming a circularly symmetric model of the galaxy, we can obtain distances to the emission at extreme velocities in the first and fourth quadrants. At each galactic longitude

$l$ , extreme velocity emission comes from the tangent point at a galactocentric distance of  $R = R_0 \sin l$  (Figure 1). The tangent points are at a distance  $d = R_0 \cos l$  from the sun. Thus the observed angular extent  $b$  gives the height  $z = R_0 \cos l \tan b$ . In the first quadrant the velocity of the gas at radius  $R$  is given by

$$V(R) = \frac{\Theta \sin l}{R/R_0} - \Theta_0 \sin l \quad (1)$$

where  $\Theta$  is the circular velocity at  $R$ . At the tangent points equation (1) reduces to  $V_T = \Theta - \Theta_0 \sin l$ .

From the observed  $b$  extent and the terminal (extreme) velocities one could calculate the scale height and the rotation velocity. The velocity profiles however do not have a sharp cutoff due to the velocity dispersion of the gas. Ideally the emission from the tangent point  $T(b,v)$  is a bivariate Gaussian in altitude and velocity; but emission from nearby radii is not well separated in velocity because of the velocity dispersion of the gas.

### 2.3. The velocity dispersion

The finite velocity dispersion has two main effects that the present analysis needs to take into account. First it makes the velocity-to-distance conversion fuzzy, so it is difficult to separate emission from  $R$  (at the tangent point) and that from  $R'$  ( $R' = R + \Delta R$ ), where  $\Delta R$  depends on the velocity dispersion. For cold gas (zero velocity dispersion) a difference in velocity  $\Delta V_T = 7 \text{ km s}^{-1}$  would correspond to  $\Delta R \simeq 270 / \sin(l) \text{ pc}$  near the tangent point. Expecting velocity dispersions of this size, it is meaningful to calculate parameters like the scale height, rotation velocity and the velocity dispersion, averaged over intervals  $\Delta R$  of a few hundred parsecs.

At the same time we must take into account the fact that the emission at velocity  $V$  comes from  $R$ , and from the annulus between  $R$  and  $(R + \Delta R)$ , so that  $T(b,v)$  is no longer a simple bivariate Gaussian. Celnik et al.(1979) derived the expression for the expected line shape near terminal velocity. In this section we calculate the 2-dimensional (in latitude and velocity) profile of emission from near the tangent point, i.e. between  $R$  and  $R + \Delta R$ , taking the scale height, rotation velocity and velocity dispersion to be constant over  $\Delta R$ . We expect the additional information in the apparent latitude-extent as a function of velocity to yield more constraints on the positions of the emitting regions.

Consider emission seen at velocities close to the terminal velocity; ( $\Delta V \ll \Theta$ )

$$V(R') = V_T - \Delta V = \Theta - \Theta_0 \sin l - \Delta V \quad (2)$$

giving

$$\frac{R'}{R_0} = \frac{\Theta \sin l}{\Theta - \Delta V} \quad (3)$$

An observer's line of sight intersects the annulus at radius  $R'$  at two points (figure 1). The distances to the points of intersection,  $r_1$  and  $r_2$  are given by

$$r_{\frac{1}{2}} = R_0 \cos l \mp R' \sqrt{1 - \frac{\sin^2 l}{(R'/R_0)^2}} \quad (4)$$

$$\Rightarrow r_{\frac{1}{2}} = R_0 \cos l \mp R_0 \sin l \sqrt{\frac{\Theta^2}{(\Theta - \Delta V)^2} - 1} \quad (5)$$

For the same scale height, gas at the subtangent points will have smaller and larger latitude extent than the tangent point gas, corresponding to the far and the near part of the annulus at  $R'$

$$\sigma_{z1}(V) = \frac{\cos l}{\cos l \mp \sin l \sqrt{2\Delta V/\Theta}} \sigma_z(V_T)$$

Finally we must take into account velocity crowding effects. The optical depth per velocity interval is determined by the line of sight distance per unit velocity interval.

$$\frac{dr}{dv} = \frac{R_0 \Theta^2 \sin l}{(\Theta_0 \sin l + V(R'))^3} \left( \frac{\Theta^2}{(\Theta_0 \sin l + V(R'))^2} - 1 \right)^{-1/2} \quad (6)$$

Thus the emission in the velocity interval  $\Delta V$  (corresponding to  $\Delta R = \Theta_0/R_0 \sin(l)$ ) near the terminal velocity, keeping parameters,  $\sigma_v$ ,  $\sigma_z$  fixed over the velocity interval is

$$T(v, z) = A \sum_{1,2} \int \frac{1}{2\pi\sigma_V\sigma_z(V)} \exp \left( -\frac{(v-V)^2}{2\sigma_V^2} - \frac{(z-z_0(V))^2}{2\sigma_z(V)^2} \right) \frac{dr}{dV} dV. \quad (7)$$

## 2.4. Fitting

The model profile is calculated (eqn. 7) as a function of latitude  $b$  and velocity  $V$  for each longitude and a least-squares fit is made to the data to determine the best fit parameters, namely the amplitude  $A$ , the centroid in the vertical direction  $z_0$ , the vertical scale height  $\sigma_z$ , the terminal velocity  $V_T$  and the velocity dispersion  $\sigma_v$ .

The fitting is done by least squares minimization using the downhill simplex routine, ‘amoeba’ (Press et al. 1993). Contour maps of emission as a function of latitude  $b$  and velocity  $V$  and superposed best-fit models are given in Appendix A (Figure 12) and Figures 2 and 3a .

The velocity range over which the fit is made is determined by the width of the extreme velocity feature. For each line of sight the spectra at all latitudes are summed to form a composite spectrum. The peak at the highest velocity is identified as the terminal velocity feature. The lower velocity at which the emission drops to half-maximum is defined as  $V_{\text{half}}$ . We fit the emission seen at velocities greater than  $V_{\text{half}}$ .

To see how sensitive the parameters obtained are to a change in the velocity range over which the fitting is done, we do the fitting for different values of  $V_{\text{half}}$  for one of the longitudes  $l = 39$ . Figure 3a shows the various values of  $V_{\text{half}}$  in relation to the terminal velocity feature. The best fit parameters for each of the fits is plotted against  $V_{\text{half}}$  in figure 3b. We see that the parameters are fairly insensitive to the velocity range over which the model is fit.

## 2.5. Error estimation

The above analysis holds true for a diffuse medium. For molecular clouds we might interpret  $T(b, V)$  as a smoothed probability of finding molecular clouds at galactic latitude  $b$  and velocity  $V$ . The molecular gas is distributed as clouds and cloud complexes, which sample this probability. The main uncertainty in deriving the parameters of the models is the shot noise due to the small number of clouds and the clumpiness of the clouds. From figures 2, 3 and 12, it is clear that the models do not trace the detailed structure of the gas.

If we ignore the clumpiness of the molecular gas layer and treat the emission as if it were from a smoothly distributed medium, the  $\chi^2$  per degree of freedom is typically found to be a few (2-19). The  $\chi^2$  per degree of freedom, or reduced  $\chi^2$  is calculated by the relation

$$\chi^2 = \frac{1}{N} \sum \left( \frac{T(b, V) - T_{\text{model}}}{T_{\text{photon}}} \right)^2 \quad (8)$$

where  $T_{\text{photon}}$  is the photon noise, and  $N$  the number of degrees of freedom. Since the main source of noise is the clumpiness of the medium, which is greater than the instrumental noise, we do not expect the  $\chi^2$  to be 1. On the other hand we use the relation  $\chi^2 = 1$  to estimate ‘realistic’ noise ‘ $T_{\text{real}}$ ’ of the emission and then proceed to calculate the confidence intervals on the parameters using the rescaled noise.

While setting the  $\chi^2 = 1$ , we have lost information on the goodness of fit of the models. Also ‘ $T_{real}$ ’ is not necessarily normally distributed, an assumption made while estimating confidence levels on the distribution parameters. So we carry out Monte-Carlo simulations of the cloud population in the galaxy. This is also useful for detecting systematic errors, to the extent that we understand and can simulate the cloud population.

If we assume the clouds to be ballistic particles, it is the locations and velocities of the clouds that follow the distribution derived in the last section. Clearly a large number of optically thin entities would mimic a smooth distribution. The velocity crowding ( $dr/dv \rightarrow \infty$ ) at the tangent points somewhat justifies the assumption that there is a large number of clouds. Velocity intervals of few  $\text{km s}^{-1}$  near the terminal velocity correspond to a kpc or so of distance  $r$  traversed along the line-of-sight. We would expect  $\sim 40$  clouds in the velocity interval of  $7 \text{ km s}^{-1}$  near the terminal velocity (at longitude  $l = 35^\circ$ ). As for ‘transparency’, molecular clouds show optically thick clumps and filaments with a typical surface filling fraction of 30% (e.g. Bally et al. 1987). Thus the approximation that we are counting the number of clouds/clumps, and that little emission from clouds is shadowed, is perhaps not entirely unjustified.

To test the limitations of this approximation, and to get an estimate of the confidence levels on the parameters, we simulate the cloud population in the Galaxy, taking the total mass of the molecular gas in the galaxy to be  $2 \times 10^9 M_\odot$ , and the proportionality constant  $X = \frac{N(H_2)}{I(CO)}$  to be  $3 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1}$ .

With the additional input of the size-linewidth-mass distribution,

$$\left( \frac{M}{100 M_\odot} \right) = \left( \frac{\Delta v}{0.36 \text{ km s}^{-1}} \right)^4 = \left( \frac{R}{\text{pc}} \right)^2 \quad (9)$$

and the mass spectrum of the clouds (Solomon & Rivolo 1989)

$$\frac{dn(M)}{dM} \propto M^{-1.5} \quad (10)$$

we can perform Monte-Carlo simulations of the cloud distribution in the galaxy. The upper and lower cutoffs for the cloud mass are taken to be  $10^6 M_\odot$  and  $10^3 M_\odot$  respectively. The cloud centers are distributed in  $(R, \theta, z)$ . The radial distribution of the gas is taken from earlier galactic surveys, which closely sample in longitude (Sanders et al. 1986, Dame et al. 1987). The azimuthal distribution at a particular radius is taken to be uniform. In the  $z$ -direction the clouds centers follow a Gaussian distribution with mean  $z = 0 \text{ pc}$ , and scale height  $\sigma_z = 50 \text{ pc}$ . The cloud-cloud velocity dispersion is taken to be  $\sigma_v = 7 \text{ km s}^{-1}$ . The clouds themselves are represented as Gaussians in  $z$  and  $v$ , with the aforementioned size-linewidth relationship between  $\sigma_z(\text{cloud})$  and  $\sigma_v(\text{cloud})$ .

The clouds are then ‘observed’ with a beam of FWHM  $100''$  for 9 longitudes spanning the latitude range of the survey. The simulated data sets have the same latitude and velocity coverage as the survey and were modeled in the same fashion. Cloud-cloud shadowing near the terminal velocity is considerable ( $\simeq 7$  for mid-longitudes) and is handled by scaling down the contribution from the shadowed region of a cloud by  $f = (1 - 0.3)^n$  where  $n$  is the number of clouds between that cloud and the sun. Detailed modeling of the beam shape, cloud sizes, radiative transfer etc. has not been carried out. A more complicated model is perhaps not justified in view of the severe blending of the features at the tangent point.

### 3. RESULTS

#### 3.1. Results of the simulations

Examples of contour maps (in b-v) derived from the Monte-Carlo simulations are shown in Figure 4 for longitudes  $l = 22.02^\circ$ ,  $l = 50.8^\circ$  and  $l = 35.1^\circ$  and the best fit models. At  $l=35.1^\circ$  the terminal velocity regions are severely crowded with typically 40 clouds in the region we are trying to fit. The cloud-cloud shadowing goes up to 7, i.e. we might be looking ‘through’ up to 7 clouds near the tangent point velocities.

The parameters resulting from fitting the models to the simulations of 9 longitudes are shown in Table II and figure 5. All the longitudes simulated have identical input distribution parameters, to test for systematic errors as the velocity crowding changes with longitude. The parameters measured agree well with the input parameters, with the exception of the scale height  $\sigma_z$ , which is measured to be higher than the input. It is reassuring that the  $\chi^2$  per degree of freedom for the simulated models is found to vary between 6 and 13, similar to the values for the observations (2-19). The velocity dispersion  $\sigma_v$  has an uncertainty of  $\pm 3 \text{ km s}^{-1}$ , while the error bars obtained from leastsquare fitting and scaled up noise correspond to  $\pm 3.5 \text{ km s}^{-1}$ . The error-bars for  $\sigma_z$  from Monte-Carlo simulations are larger than the ones from the leastsquare analysis. There are no systematic trends in the estimates of the parameters with longitude.

The scale height of the gas is expected to be higher than the scale height of the cloud centers, since  $\sigma_{\text{centers}}$  and  $\sigma_{\text{cloud}}$  add in quadrature to give the scale height of the gas. We repeat the simulations with clouds of different sizes at  $l=35.1^\circ$ , and the measured scale height of the gas does go down with the size of the clouds. The values of  $\sigma_z$  however are still slightly higher than expected if the  $\sigma_{\text{centers}}$  and  $\sigma_{\text{cloud}}$  add in quadrature. This is because of crowding/shadowing which tends to flatten the z-distribution. Simulations

with no shadowing (i.e. transparent clouds) and all of the mass in  $10^4 M_\odot$  and  $10^3 M_\odot$  clouds reproduce the expected scale height of the gas (cf Table III). For the standard mass-spectrum of the clouds in  $10^6 M_\odot$  to  $10^3 M_\odot$  clouds the contribution to scale height defined by  $\sigma_z^2(\text{cloud}) = \sigma_z^2(\text{measured}) - \sigma_z^2(\text{centers}) = 56 \text{ pc}$ . Only one longitude in the Knapp et al. survey shows a scale height larger than 55 pc (table I), making it difficult to accommodate the size spectrum used for the simulations.

The scale heights estimated for the Monte-Carlo simulations depend on other properties like the density profiles of the clouds. Using clouds with a sharp cutoff in emission at the cloud boundaries as seen in local clouds (e.g. Blitz & Thaddeus 1980), the measured scale height is lower than estimated for clouds with Gaussian density profiles (table II). Simulations of emission at  $l=35.1^\circ$  were done with different values of scale height  $\sigma_z(\text{centers}) = 2, 10, 20, 30$  and  $40 \text{ pc}$ , and a size spectrum of  $10^3 - 10^6 M_\odot$ . The contribution of the cloud size to the scale height  $\sigma_z^2(\text{cloud})$  is constant  $\simeq 42 \pm 2.7 \text{ pc}$  (figure 6). The scale height of molecular gas at the tangent points is observed to be  $< 40 \text{ pc}$  along many lines of sight. If the resulting scale height of the simulated data sets were not sensitive to cloud properties we would conclude that this analysis favors small clouds.

Since the molecular gas is clumped, we wish to investigate the effect of having no gas at the tangent points. It has been shown by Shane & Beiger-Smith (1966) that tangent points with little or no gas show a low, broad-shouldered profile (also see figure 7). For the distribution of clouds a lower profile may be taken to mean that statistically there will be fewer clouds at the extreme velocities. Since we allow the fitting program to choose the velocity dispersion  $\sigma_v$  and the terminal velocity  $V_T$ , missing tangent point gas will lead to a lower value of  $V_T$  and a higher value of  $\sigma_v$ . The other possibility is that the fitting routine will get confused by the small number of clouds and fit a single cloud giving a small  $\sigma_v$ . The scatter in the parameters is found to be fairly large for the simulations with no gas at the tangent points, as expected from a smaller number of clouds (figure 8). Thus the very high ( $11 \text{ km s}^{-1}$ ) as well as the very low ( $2 \text{ km s}^{-1}$ ) values of  $\sigma_v$  measured from the data are not believable.

### 3.2. Distribution parameters

So far we have derived the general functional form of  $T(b,V)$  and the shape of the emission contours. We also see that the derived parameters are robust to small changes in the fitting procedure, and the derived uncertainties in the parameters are comparable to those from the simulations. All the data and the best fit models are shown in Appendix A, figure 12.

Figures 9 and 10, and table 1 show the distribution parameters  $V_T, \sigma_v, \sigma_z, z_0$  for all the longitudes. At low longitudes ( $l=17^\circ, 19^\circ$ )  $\sigma_v$  and  $\sigma_z$  are found to have low values but with larger error-bars, due to the relatively small amount of CO at those radii.

The midplane position  $z_0$  as derived from the fitting shows a smooth undulation (figure 10). The scale height increases with radius  $R$ ; also the errors in  $\sigma_z$  increase, mainly due to the coverage in latitude translating to a smaller coverage for the nearby gas at higher longitudes. A linear fit gives a gradient in scale height of  $5.5 \text{ pc kpc}^{-1}$ .

The terminal velocity closely matches the terminal velocity curve for HI (Gunn et al. 1979), pointing to the similarity of the HI and CO distribution and kinematics. This also reassures us that the tangent points are not substantially devoid of molecular gas, since HI has been shown to be present at tangent points by the absence of low broad-shouldered profiles (Shane and Beiger-Smith 1966).

The values of the velocity dispersion found for different lines of sight range from  $\sim 2 \text{ km s}^{-1}$  to  $11 \text{ km s}^{-1}$ . There is however a trend for the  $\sigma_v$  to increase at higher longitudes, i.e. with increase in galactocentric radius. A straight line fit through the  $\sigma_v$  values shows an increase from  $3.8 \text{ km s}^{-1}$  at  $2.5 \text{ kpc}$  to  $7.1 \text{ km s}^{-1}$  at  $7.5 \text{ kpc}$ .

### 3.3. Midplane mass density of the disk

Considering the condition for vertical equilibrium of the clouds in the galactic disk, in the simplest case of a single isothermal population of clouds, the vertical component of the gravitational force is balanced by the pressure due to turbulent motions of the gas  $P_z = \rho_z \sigma_v^2$ . The thickness of the molecular gas layer being much less than that of the mass, the vertical component of the force  $K_z = -4\pi G \rho_0 z$  ( $\rho_0$  is the midplane mass density) and the distribution of gas is a Gaussian with scale height  $\sigma_z$ . The midplane mass density of the disk is then given by

$$\rho_0 = \frac{\sigma_v^2}{4\pi G \sigma_z^2} \quad (11)$$

With the additional assumption of isotropy of velocity dispersion of clouds, we can estimate  $\rho_0$  from the scale height and the velocity dispersion (Knapp 1988).

Figure 11 shows the midplane mass density of the disk computed from equation 10 and the best fit parameters  $\sigma_v$  and  $\sigma_z$  for different longitudes. We fit an exponential disk  $\rho = \rho_c \exp(-R/R_c)$ , yielding a scale length  $R_c = 5 \text{ pc}$  and midplane mass density at the

center of the disk  $\rho_0(0) = 1.1 M_\odot \text{pc}^{-3}$ . From these two values we get the midplane mass density at sun  $\rho_0(R_0) \simeq 0.2 M_\odot \text{pc}^{-3}$ . The scale lengths of 3.2 kpc and 10.1 kpc and central densities of  $\rho_0(0) = 0.5 M_\odot \text{pc}^{-3}$  and  $0.07 M_\odot \text{pc}^{-3}$  lie within 66% confidence limits, leading to an uncertainty of a factor  $\sim 2.5$  in the value of  $\rho_0(R_0)$ .

The simulations however show that the parameters determined have systematic errors. The scale height of the gas is measured to be systematically higher than the scale height of the cloud centers. If we knew the sizes of the molecular clouds reliably we could compensate for that. Liszt and Burton (1981) favor a smaller (5-10 pc) typical cloud size by comparing the morphology of the l-v diagrams of simulations with observed data. If most of the mass of the molecular is in small clouds the corrections to the scale height are small. Shadowing of one cloud by another flattens the distribution and leads to a further overestimate of the scale height. This effect should be smaller for higher longitudes (less crowding/shadowing); so we probably are overestimating the scale length  $R_c$ . Apart from these effects seen in the simulations, we may expect the velocity dispersion measured to have a non-random component from streaming motions as we are measuring the horizontal velocity dispersion. Further discussion of this effect is postponed to the next section.

#### 4. DISCUSSION

For a consistent analysis of the vertical equilibrium in the disk, we must measure the scale heights and the velocity dispersion of the *same* population. In the current analysis we do just that for the molecular gas seen at the tangent points. The tangent point emission is modeled in two dimensions, so the scale height and the velocity dispersion are fit simultaneously, along with the other two free parameters of the model, the rotation velocity and the centroid of the z-distribution. One might worry about correlated errors in the determination of the four quantities, for example a lower terminal velocity estimation may lead to a higher measurement of the velocity dispersion. While we don't know the real parameters we can estimate the local dips and peaks in the parameters as compared to the values smoothed over large radii. The fluctuations in different parameters do not seem to be correlated (figures 9 and 10). Moreover the best-fit parameters of the Monte-Carlo simulations, where we do know the actual (i.e. input) parameters, do not show correlated errors.

We are measuring the scale height and the velocity dispersions for the tangent point gas. The fact that we are sampling a portion of the total gas in the inner galaxy may be of concern in that the parameters may not be very representative. The velocity crowding at the tangent points means that we are typically looking at  $\simeq 1$  kpc of path length. With

the present analysis we fit the parameters to intervals of a few hundred parsecs in Galactic radius, about 1 kpc in the line of sight distance. The typical number of clouds at tangent points is 40 (at longitude  $l = 35^\circ$ ), so we are not finding the parameters of just a single cloud unless the tangent point regions are severely underpopulated.

#### 4.1. Tangent point analysis

Since we expect the velocity dispersion to have values of few  $\text{km s}^{-1}$ , the tangent point emission has to be modeled more carefully than if we were seeking to derive the rotation curve. Most of the analyses of the tangent point gas have been done to determine the rotation curve of the gas (atomic or molecular). In this paper we are concerned more with the determination of velocity dispersion and the scale height of the gas. In principle the velocity dispersion and scale height could be estimated simply by looking at the gas at forbidden velocities, i.e.  $V > V_T$ . This is not very practical however for the following two reasons. The first has to do with the calculation of the terminal velocity; most estimates involve assumptions about the value of the velocity dispersion, or at least its constancy (Gunn et al. 1979), or the size-linewidth relation of the constituent clouds (Clemens 1985, Liszt et al. 1981). It would be quite circular then to use such a rotation curve to calculate velocity dispersions. One could use the rotation curve determined from HI data to determine the velocity dispersions for molecular gas. The disadvantage in doing so is mainly due to the clumped nature of molecular gas, which means we see only a few clouds at forbidden velocities. For this reason the velocity dispersion and scale heights found by this method are very sensitive to the terminal velocity. Shane and Bieger-Smith (1966) and Burton and Gordon (1978) subtract the equivalent width of the terminal velocity feature, which overestimates the velocity dispersion, and underestimates the terminal velocity. These methods give values of terminal velocities accurate to a few  $\text{km s}^{-1}$  - about the value expected for the velocity dispersion.

#### 4.2. Velocity dispersion

Various methods have been used to estimate the velocity dispersion of molecular gas. These are summarized by Stark & Brand (1989). The values of  $\sigma_v$  found are also diverse, from  $4.2 \text{ km s}^{-1}$  to  $7.8 \text{ km s}^{-1}$ . The situation is further complicated by the definition of velocity dispersion, and its possible confusion with streaming motions. As a working definition we assume that any motion that averages out for a distance of  $\Delta R = 200 - 400 \text{ pc}$  does not contribute to the velocity dispersion. This is achieved by keeping the parameters

(for example rotation velocity  $\Theta$ ) stiff over such intervals in radius, while fitting the terminal velocity feature (section 2.3). In treatments where the terminal velocity is allowed to vary between longitudes quite close to one another ( e.g. figure 3 in Burton & Gordon (1978)) smoothing the rotation curve to  $\Delta R = 200 - 400$  pc and calculating the scatter of  $V_T$  around the smoothed value would be a method equivalent to ours.

Stark (1984) and Stark and Brand (1989) estimate the velocity dispersion of a population of local clouds, at distances of up to 3 kpc from the sun. They find a velocity dispersion  $\sigma_v = 7.8^{+0.6}_{-0.5} \text{ km s}^{-1}$ . Given that they do not distinguish between streaming and random motions, and have an operational definition similar to ours; their value of  $\sigma_v$  found in the solar neighborhood is consistent with our value.

The  $\sigma_v$  determination likely to be most free of the vagaries of the rotation curve and large scale motions is the velocity dispersion derived for high latitude clouds by Blitz et. al. (1984) who find  $\sigma_v = 5.7 \pm 1.2 \text{ km s}^{-1}$  for a sample of 28 clouds, after excluding a ‘pathological cloud’ with velocity  $-24.6 \text{ km s}^{-1}$ . This is however a measure of the velocity dispersion (mostly) in the vertical direction which may be different from the line-of-sight velocity dispersion estimated in this paper. Note that these are smaller clouds, and if they have a velocity dispersion greater than the galactic plane GMCs (Stark 1983) the vertical velocity dispersion value may be still smaller than the velocity dispersion in the plane.

### 4.3. Vertical distribution of the gas

The vertical distribution of the gas is found in this study is consistent with previous values. The scale height is found to increase with radius. A linear fit gives a gradient in scale height of  $5.5 \text{ pc kpc}^{-1}$ , and extrapolates to give a scale height 58 pc at  $R_\odot$ . Similar values are found by Dame and Thaddeus (1985), Dame et. al. (1987), Sanders et. al. (1984), Grabelsky et. al. (1987), and Clemens et. al. (1988). We find from the Monte Carlo simulations that the (cloud-cloud) scale height is overestimated if the cloud sizes are comparable to the scale height, and because of cloud-cloud shadowing at tangent points. The second effect is expected to be smaller at tangent points seen at higher longitude. This introduces an error in the gradient of the scale height with radius. The first effect (cloud size) will lead to an incorrect constant term in the scale height-radius relation. Extrapolating the  $\sigma_z$  to  $R_0$  we may under or overestimate  $\sigma_z(R_0)$  depending on which of these effects dominates. Local determinations of the scale height at  $R_0$  for clouds/HII regions (Fich & Blitz 1984) give a value of 88 pc. The distinction between the distribution of cloud centers and the scale height of the gas may be best made for local clouds.

The positions of the CO layer centroid,  $z_0$ , are also consistent with the above mentioned

studies. The centroid  $z_0$  also shows a dip, going as far as  $\simeq 50$  pc (comparable to  $\sigma_z$ ) south of the midplane. Simulations made with the distribution of gas centered at  $z=0$  do not show large deviations from  $z=0$  (figure 5, table II). This feature is not seen in the studies of the vertical distribution of HI (Celnik et al. 1979). Unfortunately it is difficult to see the azimuthal structure of this waviness, since we can only look at the tangent points. At any rate this smooth variation argues against massively incorrect velocity-to-distance transforms, since that would disturb the smoothness of such a feature if it was there, and would require a conspiracy to produce one if it were not.

#### 4.4. Mid-plane mass density of the disk

The mid-plane mass density of the disk has been derived making numerous assumptions. We assume that the molecular gas layer is in equilibrium, at least on the larger scales. We also assume that there is a single population of molecular clouds at least with respect to their velocity dispersions. There may be more than one population of molecular clouds with different velocity dispersions. The vertical distribution of molecular gas in that case will not be a single Gaussian but a sum of more than one Gaussian. One way to test these assumptions is to see whether the vertical distribution is indeed Gaussian. We show in a subsequent paper that there exists a population of small clouds at high  $z$ . The tail of the distribution is inconsistent with a Gaussian distribution. Similar high tails have been found for HI (Lockman 1984). Given the ‘noise’ in the data due to the small number of molecular clouds, we do not think a more complicated model can be derived.

We also assume that the clouds are independent ballistic particles so the number of clouds found in any region is a Poissonian variable. Moreover it is assumed that the distribution of cloud centers is dictated by the dynamics of the disk alone. In fact the occurrence of a cloud in a region of space is not a Poissonian event as the clouds are correlated in space and organized into cloud complexes.

To examine the vertical equilibrium the value of the *vertical* velocity dispersion should be known, something possible only for local molecular clouds. We are *always* measuring the *azimuthal* velocity dispersion when we look at the tangent points. The gas being a dissipational component we expect the velocity dispersion to be isotropic. The assumption of the isotropy of the velocity dispersion can be checked by comparing the  $\sigma_v$  derived for high latitude clouds (in part the vertical component), which is  $5.7 \pm 1.2 \text{ km s}^{-1}$  (Blitz et al. 1984), with the value of the azimuthal component (extrapolated value at the solar neighborhood), which we find to be  $\simeq 7.8 \text{ km s}^{-1}$ . Within the errors this value is consistent with the velocity dispersion for the high latitude clouds and also with the expected shape

of the velocity ellipsoid.

The values of the radial ( $u$ ), azimuthal ( $v$ ) and the vertical ( $w$ ) velocity dispersion of (late type) stars in the solar neighborhood are related as:  $u \simeq 2w$ ,  $v \simeq \sqrt{2}w$  (Mihalas & Binney 1981). Given the uncertainty in the estimated velocity dispersion we cannot determine if the velocity ellipsoid of the gas has that shape, even locally. Support for the isotropy of the velocity dispersions comes from the shape of the velocity ellipsoid of early type stars. The O and B stars show fairly isotropic velocity dispersions (Mihalas & Binney 1981, p 423);  $\langle v^2 \rangle / \langle u^2 \rangle \simeq 1$ , although the vertical component  $\langle w^2 \rangle$  is consistently but slightly lower. These stars are young enough that they show mainly the kinematics of the interstellar medium.

We would like to check the mass density profile estimated in this study against previous measurements; two such check points are the scale length on the exponential disk (of both the light and mass distribution), and the mass density in the solar neighborhood. Using the values of velocity dispersion and the scale height extrapolated to solar radius we get a  $\rho_0(R_0) = 0.34 M_\odot \text{pc}^{-3}$ ; extrapolation of the exponential profile fitted to individual  $\rho_0(R)$  gives  $\rho_0(R_0) = 0.2 M_\odot \text{pc}^{-3}$  with a formal uncertainty of a factor of 2.5. Using the local value of vertical velocity dispersion  $\sigma_v$  (Blitz et al. 1984) gives  $\rho_0(R_0) = 0.18 M_\odot \text{pc}^{-3}$ . These results are consistent with the local mass density determinations from stellar kinematics  $\rho_0(R_0) = 0.2$  (Bahcall et al. 1992, Bahcall 1984a, Bahcall 1984b, Bahcall 1984c) and  $\rho_0(R_0) = 0.1$  (Bienaymé et al. 1987, Kuijken & Gilmore 1989a, Kuijken & Gilmore 1989b, Kuijken & Gilmore 1989c, Kuijken & Gilmore 1991a, Kuijken & Gilmore 1991b, Kuijken 1991). The best fit exponential mass distribution gives a scale length 5 kpc, again with an uncertainty of a factor of 2. The scale length of the light in the galactic disk has been found to be 3.4 kpc (de Vaucouleurs & Pence 1978) and 5 kpc (van der Kruit 1987). Both the scale length and the local  $\rho_0(R_0)$  found in the present study are consistent with the values of 5 kpc and  $0.1 M_\odot \text{pc}^{-3}$  estimated by Merrifield (1992).

The systematic errors in this study as we know them are as follows. The vertical velocity dispersion may be lower than the azimuthal velocity dispersion used by a factor of  $\sqrt{2}$ . This would lower the mass density estimate from the one given here by a factor of two. The scale height may be overestimated here because of cloud size and also because of shadowing of clouds by one another. This could raise the mass density estimate from the one calculated here by a factor of at least 2.8. Also it may lead to an overestimate of the scale length of the mass distribution since the shadowing is lower at high longitudes. Given the systematic and statistical errors none of the parameters of the mass distribution mentioned in the last paragraph is favored over the other

## 5. CONCLUSIONS

In this work we have modeled the emission from molecular gas at the tangent points in the first quadrant of the Galaxy and measured the terminal velocity, the line of sight velocity dispersion, the scale height and the deviation from the galactic plane. The modeling takes into account emission from a large path along the line of sight to the tangent point. The errors in the parameters are estimated by applying the modeling technique to simulations of the galactic molecular cloud population. We find

(1) The terminal velocities are in good agreement with the terminal velocities from HI data, indicating that there is molecular gas at most terminal points, and once again pointing to the similarity of the kinematics of atomic and molecular gas.

(2) The velocity dispersion of the molecular gas increases with radius between  $R = 2.5$  kpc and  $7.5$  kpc. A straight line fit extrapolates to give a local value of  $7.8 \text{ km s}^{-1}$ .

(3) The scale height also increases with the galactic radius. Again a linear fit to the scale-height as a function of radius gives a local value of  $57 \text{ pc}$ .

(4) The centroid of the a dip below the galactic plane with a maximum excursion of  $-51 \text{ pc}$  at  $R/R_0 = 0.7$ .

(5) The simulations of the distribution of molecular clouds with ‘standard’ properties show morphology similar to the data. Using these simulations to estimate the errors in the parameters, we see that the scale height of the gas is measured to be higher than the scale height of the cloud centers, depending on the size of the clouds. This effect cannot be corrected for without better knowledge of cloud properties. The simulations also show that missing gas at the tangent points leads to an overestimate of the velocity dispersion.

(6) The midplane mass density of the disk shows a decline with the Galactic radius. An exponential disk model fitted to the data gives a scale length of  $5 \text{ kpc}$ . The local midplane mass density obtained from extrapolating this model is  $\rho_0(R_0) = 0.2 M_\odot \text{ pc}^{-3}$ . Both the scale length and the local mass density estimates are uncertain to a factor of two at least.

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## 6. Appendix A

This section shows the latitude-velocity maps of  $^{12}\text{CO}(J : 1 \rightarrow 0)$  emission at different galactic longitudes from the survey of Knapp et al. (1985) along with the best fit models for each line of sight. The latitudes have been converted to the height above the midplane at tangent points, according to the relation  $z = b R_0 \cos l$ , and the y axis is labeled as such. This conversion holds only for the tangent point emission at extreme positive velocity.

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Figure 1: The geometry of the tangent point emission. S indicates the position of the sun, R is the Galactic radius at the tangent point, and R' is a sub-tangent point. The models of the tangent point emission take into account the emission from the annulus between R and R'

Figure 2: Latitude-velocity maps of  $^{12}\text{CO}$  emission at the longitude  $l=22^\circ$  (contour levels: 1.6, 2, 3, 4, 6, 8, 10, 12 K). The best fit model to the tangent point emission is shown superposed on the data. The model shows an abrupt cutoff at the velocity  $V_{\text{half}}$ , because the fitting is done only for  $V > V_{\text{half}}$ ;  $V_{\text{half}}$  being the velocity at which the tangent point emission is half its peak value (cf. Section 2.4).

Figure 3(a): Latitude-velocity maps of  $^{12}\text{CO}$  emission at the longitude  $l=39^\circ$  (contour levels: 1.6, 2, 3, 4, 6, 8, 10, 12 K) along with the best fit model. The vertical lines show the various values of  $V_{\text{half}}$ . The fitting is done for emission at velocities  $V > V_{\text{half}}$ .

Figure 3(b): The best fit parameters of the model for the longitude  $l = 39$ : the terminal velocity  $V_T$ , the velocity dispersion  $\sigma_v$ , the centroid in  $z$  -  $Z_0$  and the scale height  $\sigma_z$  are plotted as a function of the cutoff velocity  $V_{\text{half}}$ . Emission at velocities  $> V_{\text{half}}$  was modeled. This figure shows that the parameters of the model are not very sensitive to small changes in the boundaries of the region they are fit.

Figure 4(a), 4(b) and 4(c): Monte-Carlo simulations of b-v emission maps at longitudes  $l=22.02^\circ$ ,  $l=35.1^\circ$  and  $l=55.59^\circ$  respectively. The morphology may be compared to the observations at the corresponding longitudes (Figure 12). Ten simulated b-v maps each for these and other longitudes ( $l=25^\circ, 28^\circ, 32^\circ, 39^\circ, 42^\circ, 46^\circ$ ) were modeled to estimate the uncertainties in the parameters. The best-fit models are shown superposed on the simulations. The side panels show the  $z$ -profile of the tangent point gas, summed over  $V > V_{\text{half}}$ , along with the best fit model. The top panels show the composite spectrum obtained by summing the emission over all observed latitudes along with the model similarly summed.

Figure 5: The average and standard deviations of the best fit parameters for the Monte-Carlo simulations of emission seen at longitudes  $l=22^\circ, 25^\circ, 28^\circ, 32^\circ, 35^\circ, 39^\circ, 42^\circ, 46^\circ$  and  $51^\circ$ . Ten simulations were done for each longitude. The parameters are - the terminal velocity  $V_T$ , the velocity dispersion  $\sigma_v$ , the centroid in  $z$  -  $Z_0$  and the scale height  $\sigma_z$  and are plotted against  $R/R_0 = \sin(l)$ . The error bars indicate the standard deviation of the measured parameters in each sample of ten simulations. Horizontal bars show the input parameters. We see that the measured scale height is systematically higher.

Figure 6: The measured scale height for various simulations for the longitude  $l=35.1^\circ$  are plotted against the input scale height for the distribution of cloud centers (open points).

Ten simulations were done for each input scale height, and the error bars indicate the variance of the measured scale height in each sample of ten. The contribution due to the large sizes of molecular clouds is calculated as  $\sigma_z^2(\text{cloud}) = \sigma_z^2(\text{measured}) - \sigma_z^2(\text{centers})$ . The triangles show the values of  $\sigma_z^2(\text{cloud})$ , which are nearly constant and have an average of  $\simeq 42$  pc (solid line).

Figure 7: The effect of missing tangent point gas. The lower panel shows the velocity crowding  $dr/dv$  vs the velocity at the longitude  $l=35.1^\circ$  (dotted line). The dashed curve is the convolution of the  $dr/dv$  function with a Gaussian of standard deviation 7, corresponding to a velocity dispersion  $\sigma_v = 7 \text{ km s}^{-1}$ , and is the expected shape of a spectrum. We get the solid curve if there is gas missing from the tangent point. The main effect is to broaden the profile. The upper panel shows a similar effect, looking at a 2-dimensional latitude-velocity map, which is limited in latitude. The thick lines show a composite spectrum made by summing the emission in latitude.

Figure 8: The effect of missing tangent point gas. The best fit parameters for the Monte-Carlo simulations of emission seen at longitude  $l=35.1^\circ$  are seen. Triangles represent the parameters obtained when gas is missing from the tangent points, open circles represent the parameters for the same simulations but with the tangent point gas present. The best-fit parameters (especially  $\sigma_v$ ) show more scatter when there is missing tangent point gas.

Figure 9: The velocity dispersion  $\sigma_z$  and the terminal velocity  $V_T$  for the tangent point gas at different Galactic longitudes  $l$  (hence Galactic radii  $R/R_0 = \sin(l)$ ). The solid line shows the terminal velocity for HI (from Gunn et al. 1979).

Figure 10: The scale height  $\sigma_z$  and the position of the centroid of the tangent point gas  $Z_0$  at different Galactic longitudes  $l$  (hence Galactic radii  $R/R_0 = \sin(l)$ ). The midplane deviation from the plane  $Z_0$  at  $l=48.6$  is comparable to the scale height.

Figure 11: The midplane mass density  $\rho_0(R)$  for different Galactic radii.  $\rho_0(R)$  is calculated from the scale height  $\sigma_z$  and the velocity dispersion  $\sigma_v$  using equation 10 (section 3.3). The solid line shows the best fit exponential disk model with a scale length of 5 kpc.

Figure 12: Latitude-velocity maps of  $^{12}\text{CO}$  emission at different galactic longitudes (central panel). The y-axis shows the height above the plane for the tangent point gas at extreme (positive) velocities. Superposed are the best-fit models of the tangent point emission. The side panel shows the z-profile of the tangent point gas, summed over  $V > V_{\text{half}}$ , along with the best fit model. The top panel shows the composite spectrum obtained by summing the emission over all observed latitudes along with the model similarly summed. The models show an abrupt cutoff at velocities  $V_{\text{half}}$ , because the fitting is done only for  $V > V_{\text{half}}$  (cf.

Section 2.4). The contour levels for the data are: 1.6, 2, 3, 4, 6, 8, 10 and 12 K. The models are contoured at 80, 60, 40 and 30 percent of the peak temperature.

Table 1. Gas Distribution parameters

longitude (in $^{\circ}$ )	Radius ( $R/R_0$ ) (kpc)	Centroid $Z_0$ (pc)	Scale height $\sigma_z$ (pc)	Terminal velocity $V_T$ ( $\text{km s}^{-1}$ )	Velocity dispersion $\sigma_v$ ( $\text{km s}^{-1}$ )	$\chi^2$ per degree of freedom
17.46	0.300	32.44	9.97	139.0	1.8	1.6
18.97	0.325	-3.11	17.96	123.9	3.6	2.2
20.49	0.350	-4.94	26.26	121.7	2.7	3.3
22.02	0.375	34.53	31.27	116.5	3.5	2.3
23.58	0.400	17.91	43.84	105.2	6.8	13.8
25.25	0.425	21.39	30.55	107.6	3.3	6.1
26.74	0.450	-0.76	38.07	106.0	7.6	6.4
28.36	0.475	-4.48	30.84	102.9	3.4	4.3
30.0	0.500	-13.78	33.56	104.1	4.5	19.1
31.67	0.525	-4.10	49.87	108.9	4.6	9.6
33.37	0.550	-10.20	36.20	97.2	8.4	8.2
35.09	0.575	-14.69	41.29	81.4	7.1	6.8
36.87	0.600	-29.09	30.30	82.0	2.1	3.2
38.68	0.625	-33.96	49.05	84.4	4.2	5.1
40.54	0.650	-39.22	38.99	69.2	10.4	4.9
42.45	0.675	-50.96	40.88	69.5	4.2	6.3
44.43	0.700	-37.47	39.59	65.5	5.9	5.9
46.47	0.725	-23.05	40.04	59.4	7.9	2.2
48.59	0.750	-19.62	52.38	55.3	4.7	2.9
50.81	0.775	-25.41	59.06	52.3	11.2	8.4
53.13	0.800	-17.69	39.83	46.2	3.5	2.0
55.59	0.825	0.57	25.38	43.7	6.0	1.8
58.21	0.850	16.04	43.45	35.8	6.8	1.9

Table 2. Monte Carlo Simulations

Longitude ( in $^{\circ}$ )	Centroid $Z_0$ ( pc)	Scale height $\sigma_z$ ( pc)	Terminal velocity $V_T$ ( km s $^{-1}$ )	Velocity dispersion $\sigma_v$ ( km s $^{-1}$ )	$\chi^2$ per degree of freedom
Input	0.0	50.0	...	7.0	...
22.02	$-4.7 \pm 18.0$	$71.6 \pm 14.5$	$112.6 \pm 5.4$	$7.2 \pm 2.7$	6.5
25.25	$-7.7 \pm 27.0$	$83.2 \pm 16.6$	$108.1 \pm 3.2$	$7.5 \pm 2.4$	11.5
28.36	$0.2 \pm 12.2$	$75.3 \pm 12.4$	$103.6 \pm 3.9$	$5.8 \pm 2.1$	6.2
31.67	$7.6 \pm 9.3$	$74.8 \pm 20.4$	$99.5 \pm 2.9$	$6.3 \pm 1.9$	6.9
35.1	$2.9 \pm 12.7$	$74.8 \pm 26.2$	$93.1 \pm 2.9$	$8.3 \pm 2.9$	6.4
38.68	$7.4 \pm 14.6$	$77.8 \pm 33.9$	$77.2 \pm 5.3$	$5.8 \pm 3.9$	12.8
42.45	$11.2 \pm 21.3$	$78.5 \pm 30.7$	$70.4 \pm 3.9$	$4.4 \pm 2.0$	10.2
46.47	$12.1 \pm 20.9$	$82 \pm 33.3$	$59.1 \pm 5.4$	$8.7 \pm 6.6$	7.6
50.81	$-1.5 \pm 15.8$	$61.7 \pm 35.4$	$54.7 \pm 4.8$	$5.4 \pm 3.2$	9.4
average	$3.0 \pm 16.9$	$75.5 \pm 24.8$	...	$6.6 \pm 3.1$	7.9

Table 3. Monte Carlo Simulations for Different Cloud Sizes

Mass Range of the clouds $M_{\odot}$	Centroid $Z_0$ ( pc)	Scale height $\sigma_z$ ( pc)	Terminal velocity $V_T$ ( km s $^{-1}$ )	Velocity dispersion $\sigma_v$ ( km s $^{-1}$ )	$\chi^2$ per degree of freedom
Input	0.0	50.0	93.5	7.0	...
$10^3 - 10^6$	$2.2 \pm 12.3$	$74.0 \pm 25.0$	$92.8 \pm 2.9$	$8.7 \pm 3.0$	6.4
$10^5 - 10^6$	$3.5 \pm 6.8$	$81.8 \pm 13.9$	$92.9 \pm 1.6$	$9.1 \pm 4.2$	7.6
$10^4 - 10^5$	$1.6 \pm 6.4$	$71.6 \pm 16.0$	$93.4 \pm 2.3$	$8.9 \pm 3.2$	7.1
$10^3 - 10^4$	$-1.8 \pm 1.8$	$57.8 \pm 5.7$	$95.0 \pm 3.0$	$6.3 \pm 2.2$	6.3
$10^3 - 10^4$ (no shadowing)	$-2.0 \pm 3.7$	$52.2 \pm 5.5$	$94.3 \pm 2.1$	$6.5 \pm 2.1$	6.5